# Geographic Data Science - Lecture V

## Space, formally

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# Today

- The need to represent space formally
- Spatial weights matrices
  - What
  - Why
  - Types
- The spatial lag
- The Moran Plot

# Space, formally

For a statistical method to be **explicitly spatial**, it needs to contain some representation of the geography, or **spatial context**

One of the most common ways is through **Spatial Weights Matrices**

- **(Geo)Visualization**: translating numbers into a (visual) language that the human brain *"speaks better"*
- **Spatial Weights Matrices**: translating geography into a (numerical) language that a computer *"speaks better"*.

Core element in several spatial analysis techniques:

- Spatial autocorrelation
- Spatial clustering / geodemographics
- Spatial regression

# *W* as a formal representation of space

# $W$

*N x N positive matrix that contains **spatial relations** between all the observations in the sample*

$$w_{ij} = \left\{ \begin{array}{rl} x > 0 & \text{if } i \text{ and } j \text{ are neighbors} \\ 0 & \text{otherwise} \end{array} \right\}$$

$w_{ii} = 0$ by convention

*…What is a **neighbor**???*

# Types of $W$

A neighbor is "somebody" who is:

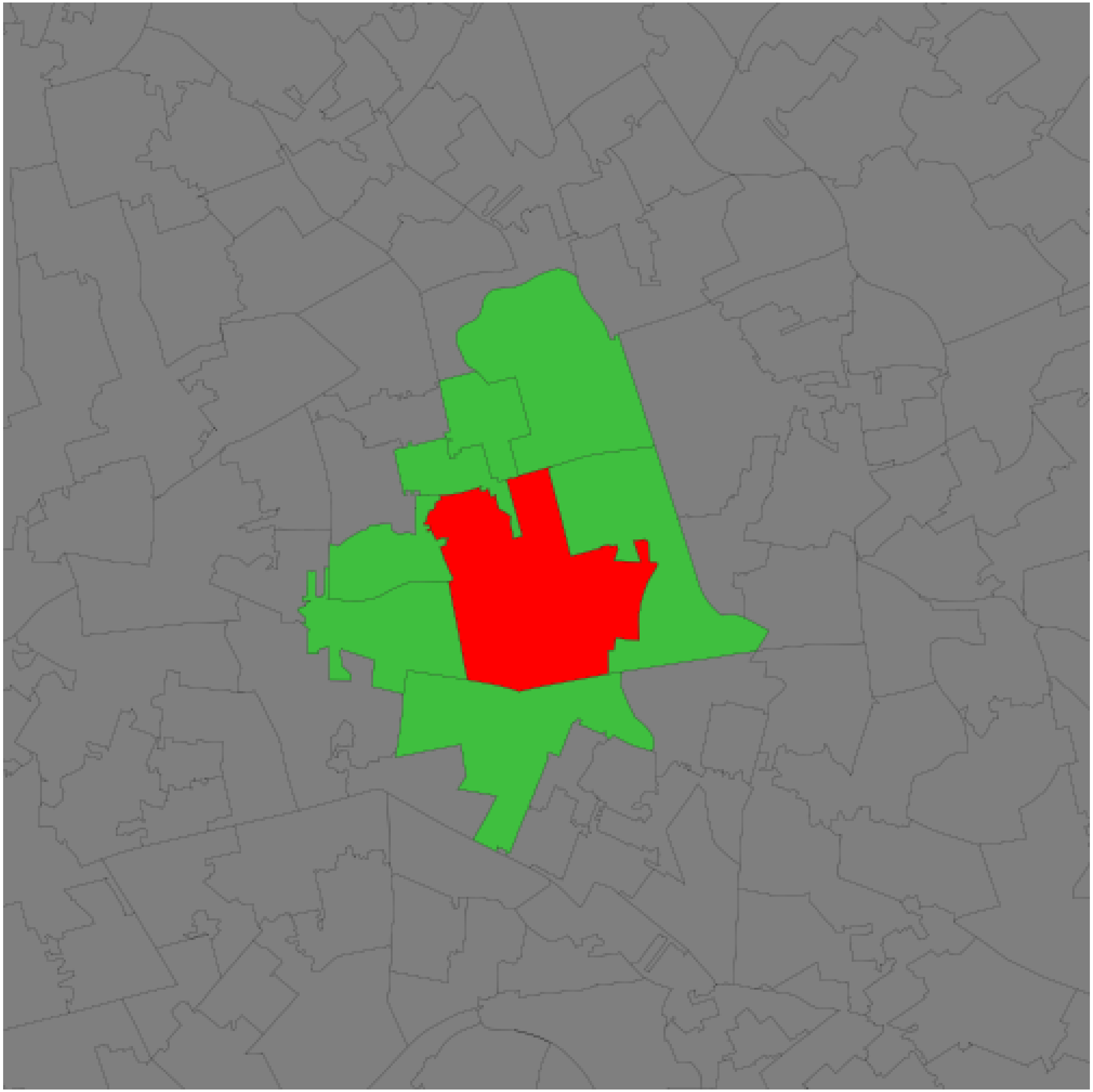
- Next door → **Contiguity**-based $W$s
- Close → **Distance**-based $W$s
- In the same "place" as us → **Block** weights
- …

See Anselin & Rey (2014) for an in-detail discussion and more types of $W$.

# Contiguity-based weights

**Sharing boundaries** to any extent

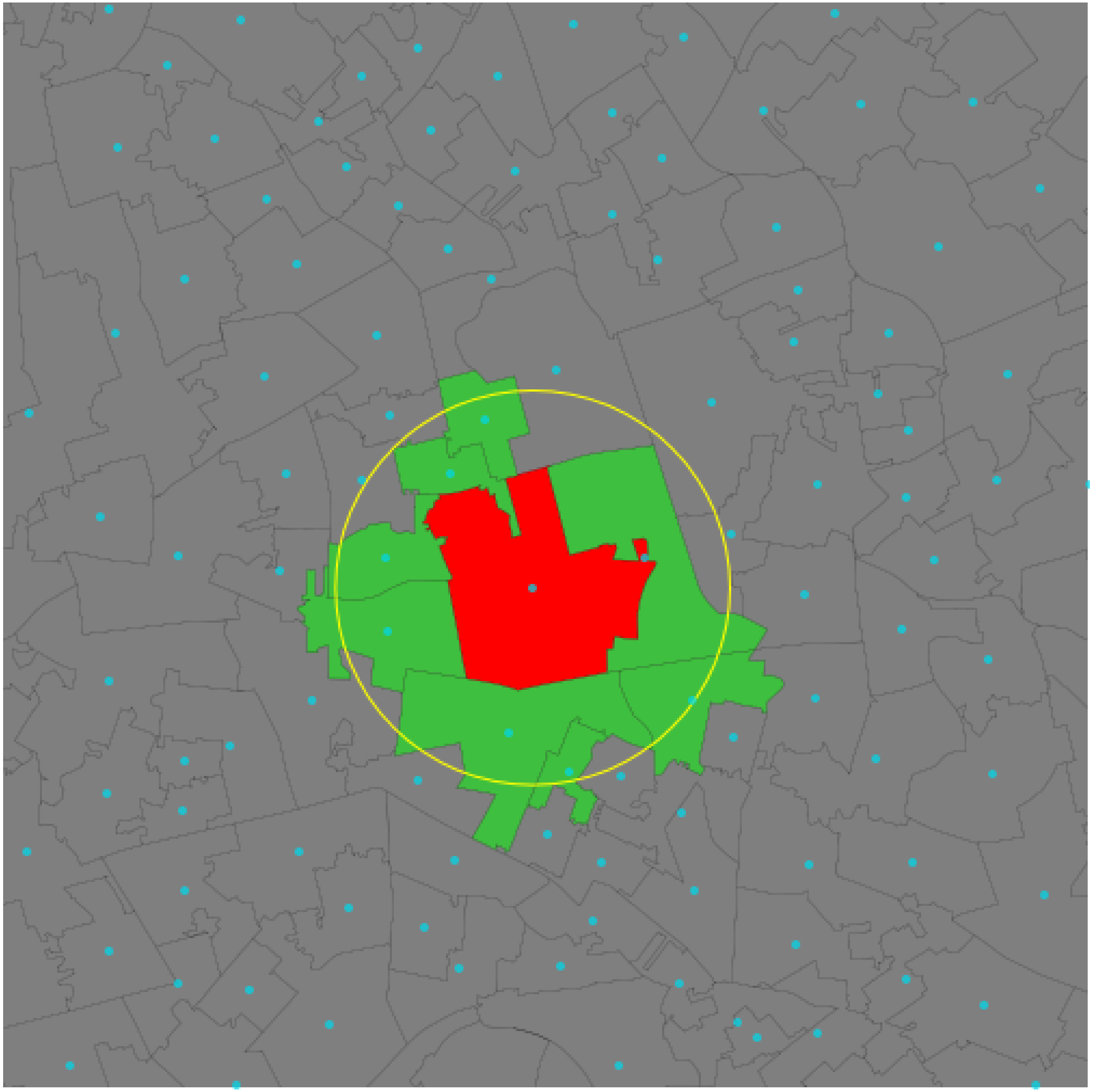
- Rook
- Queen
- …

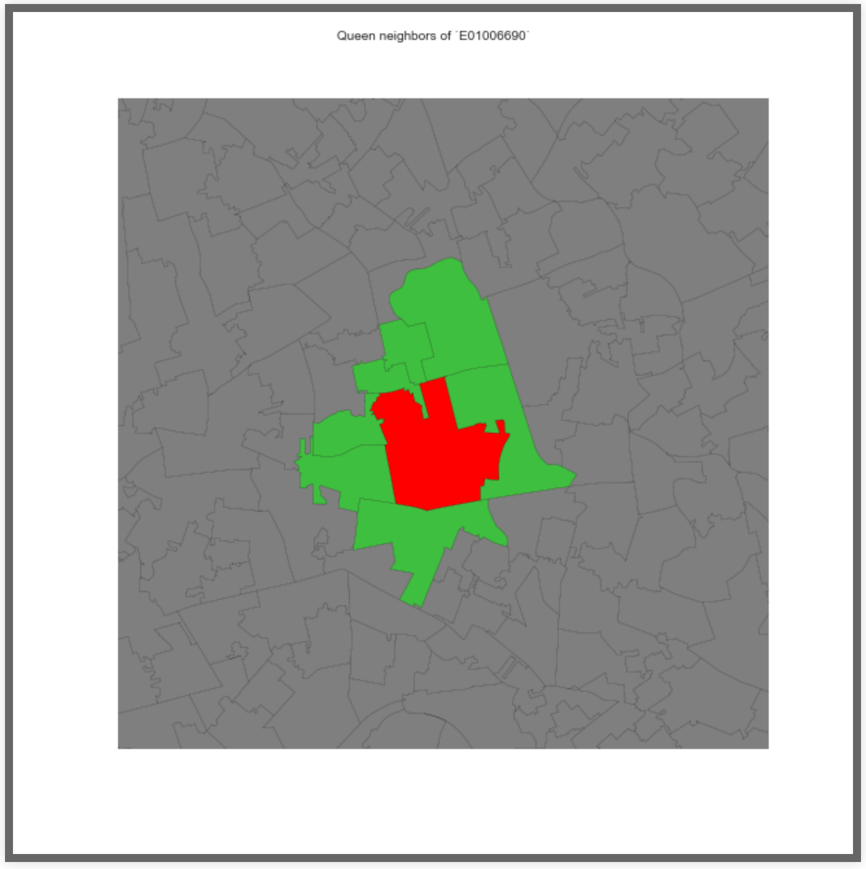


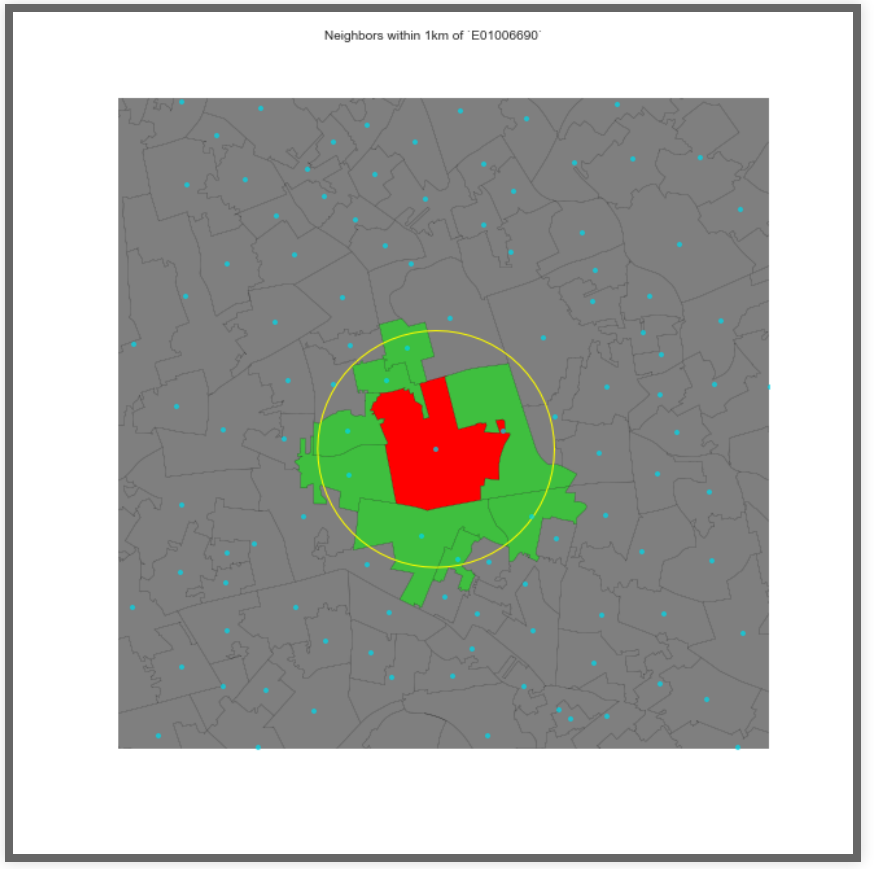
# Distance-based weights

Weight is (inversely) proportional to distance between observations

- Inverse distance (threshold)
- KNN (fixed number of neighbors)
- …




Queen neighbors of `E01006690`

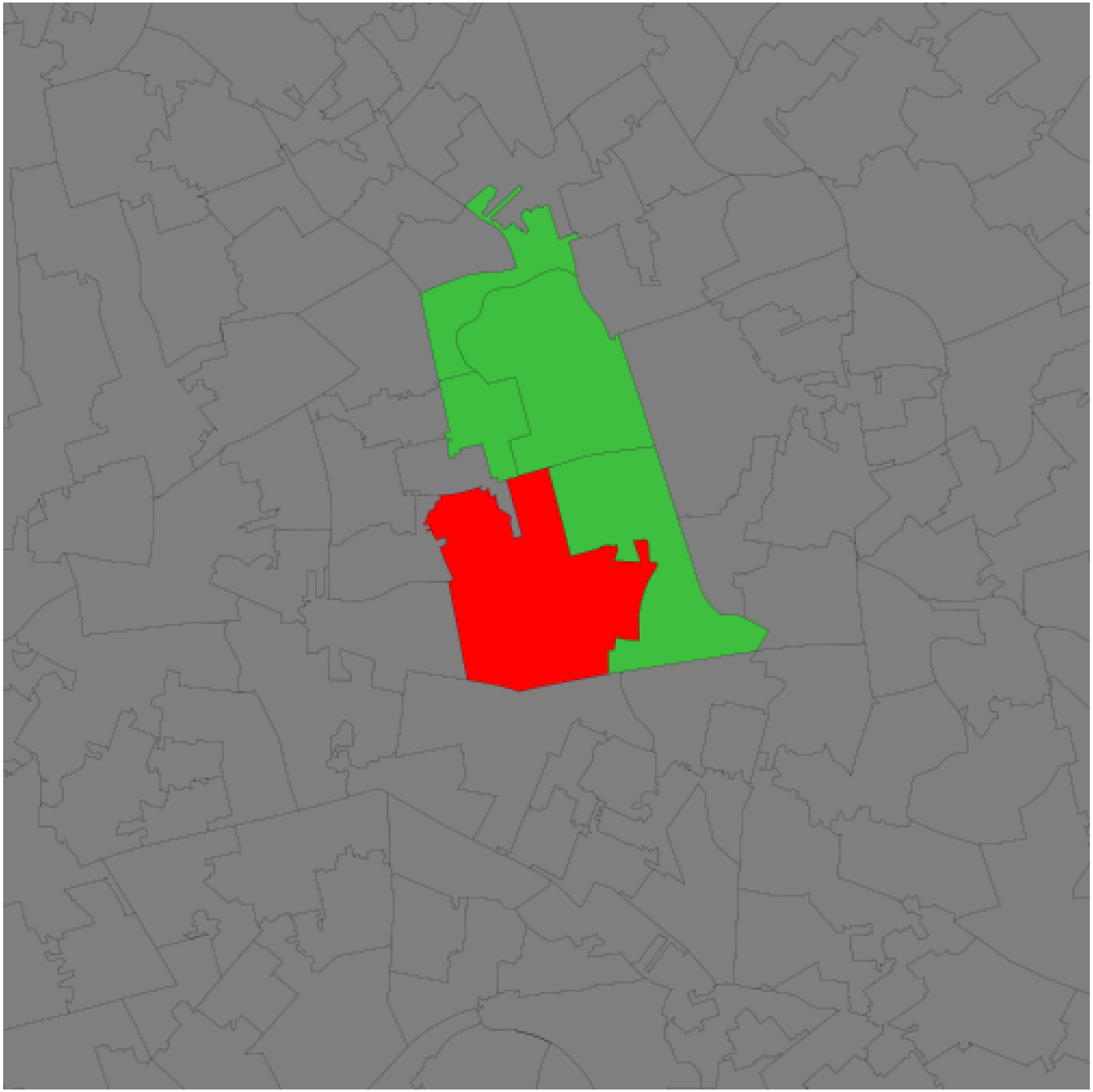

Neighbors within 1km of `E01006690`

# Block weights

Weights are assigned based on discretionary rules loosely related to geography

For example:

- LSOAs into MSOAs
- Post-codes within city boundaries
- Counties within states
- …



# *How much of a neighbor?*

No neighbors receive zero weight: $w_{ij} = 0$

Neighbors, it depends, $w_{ij}$ can be:

- One $w_{ij} = 1 \rightarrow$ Binary
- Some proportion ($0 < w_{ij} < 1$, continuous) which can be a function of:
  - Distance
  - Strength of interaction (e.g. commuting flows, trade, etc.)
  - …

# Choice of $W$

Should be based on and reflect the **underlying channels of interaction** for the question at hand.

Examples:

- Processes propagated by inmediate contact (e.g. disease contagion) → Contiguity weights
- Accessibility → Distance weights
- Effects of county differences in laws → Block weights

# Do your own (contiguity) weights time!

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 3 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 4 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 5 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 6 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 7 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 8 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| 9 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |

# Standardization

In some applications (e.g. spatial autocorrelation) it is common to *standardize W*

The most widely used standardization is **row-based**: divide every element by the sum of the row:

$$\bar{w}_{ij} = \frac{w_{ij}}{w_{i\cdot}}$$

where $w_{i\cdot}$ is the sum of a row.

# The spatial lag

# The spatial lag

Product of a spatial weights matrix $W$ and a given variably $Y$

$$Y_{sl} = WY$$

$$y_{sl-i} = \sum_j w_{ij} y_j$$

- Measure that captures the behaviour of a variable in the neighborhood of a given observation $i$.
- If $W$ is **standardized**, the spatial lag is the *average value of the variable in the neighborhood*

- Common way to introduce space formally in a statistical framework
- Heavily used in both ESDA and spatial regression to delineate neighborhoods. Examples:
  - Moran's I
  - LISAs
  - Spatial models (lag, error…)

# Recapitulation

- Spatial Weights matrices: matrix encapsulation of space
- Different types for different cases
- Useful in many contexts, like the spatial lag and Moran plot, but also many other things!

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